# Glueball masses and Pomeron trajectory in nonperturbative QCD approach

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**Abstract**

Using a nonperturbative method based on asymptotic behaviour of Wilson loops we calculate masses of glueballs and corresponding Regge–trajectories. The method contains no fitting parameters and the mass scale is fixed by the meson Regge slope. Theoretical predictions for lowest glueball states are in a perfect agreement with lattice results. The leading glueball trajectory and its relation to the Pomeron is discussed in details. Important role of mixing between glueball and $q\bar{q}$ trajectories is emphasized.

Glueballs are among the most intriguing objects both in experiment [1] and lattice data [2,3]. While experimental situation is not yet settled, lattice simulations yield an overall consistent picture of lowest ($< 4GeV$) mass spectrum. The mass scale and level ordering of the resulting glueball spectra differ strongly from those of meson spectra, yielding unique information about the nonperturbative (NP) structure of the gluonic vacuum. Another important feature of glueball spectrum is that the leading glueball Regge trajectory is closely connected to the Pomeron singularity, which determines asymptotic behaviour of high–energy diffractive processes,– the object of active studies during last decade.

In this paper we will study the problem of spectra of glueballs and of the Pomeron singularity using the method of Wilson-loop path integrals developed in papers [4,5,6]. The method is based on the assumption of an area law for Wilson loops at large distances in QCD, which is equivalent to the condition of confinement of quarks and gluons. It is shown below that long distance dynamics is defined by the string tension only and yields spin–averaged glueball masses, while spin splittings are due to mostly perturbative exchanges and relatively small. Our predictions for masses of lowest

spin-averaged glueball states in units of $\sqrt{\sigma}$ are in a perfect agreement with results of recent lattice calculations [2,3]. Spin-orbit and spin-spin splittings are calculated and found in a good agreement with lattice data also.

To find the Pomeron singularity, we calculate the leading glueball Regge-trajectory in the positive $t$ region and extrapolate it to the scattering region of $t \leq 0$. The importance of mixing among this trajectory and $q\bar{q}$–trajectories $(f, f')$ is emphasized. A calculation of these mixing effects leads to the Pomeron trajectory with $\alpha_P(0) > 1$ ($\alpha_P(0) = 1.1 \div 1.2$) in accord with experimental observations [7]. An interesting interplay of 3 vacuum trajectories in the region $t > 0$ is observed.

A previous study of a glueball spectrum by a similar method was done in ref.[8]. In this letter we improve results of [8] in several ways. First, the analysis of perturbative gluon exchanges (PGE) shows that PGE do not sum up to the adjoint Coulomb interaction, but rather to the BFKL ladder, where loop correction strongly reduces the sum, so that PGE can be disregarded in the first approximation. Secondly, we use the rotating string Hamiltonian of [5,6] which yields correct string slope $1/2\pi\sigma$ (as compared to $1/8\sigma$ in [8]). Finally the mixing of glueball and $f, f'$ trajectories is taken into account, which drastically changes the resulting Pomeron.

Following ref.[4], we separate gluonic fields $A_\mu$ into nonperturbative background $B_\mu$ and perturbative gluons $a_\mu$, $A_\mu = B_\mu + a_\mu$ and consider two-gluon glueballs, described by the Green's functions

$$G_{\mu\nu,\mu'\nu'}(x,y|x',y') = \langle \Gamma^{in} G_{\mu\mu'}(x,x') G_{\nu\nu'}(y,y') \Gamma^{out} \rangle + \text{perm} \qquad (1)$$

where $\Gamma^{(in,out)}$ are operators projecting given quantum numbers and $G_{\mu\mu'}$ is the gluon Green's function of field $a_\mu$ in the background field $B_\mu$, namely

$$G_{\mu\nu}(x,y) = \langle x|(-\hat{D}^2\delta_{\mu\nu} - 2ig\hat{F}_{\mu\nu})^{-1}|y\rangle \qquad (2)$$

where $\hat{D}_\mu = \partial_\mu - ig\hat{B}_\mu$, $\hat{F}_{\mu\nu}$ is the field strength of the field $\hat{B}_\mu$ in the adjoint representation, and averaging over background $B_\mu$ is implied by angular brackets.

Expression (1) can be written as the path integral (see refs. [4,8])

$$G_{\mu\nu,\mu'\nu'}(x,y|x',y') = const \int_0^\infty ds \int_0^\infty ds' Dz Dz' e^{-K-K'} \langle W_F \rangle \qquad (3)$$

where $K = \frac{1}{4}\int_0^s (\frac{dz}{d\tau})^2 d\tau$, $K'$ is the same with primed $z, \tau, s$, and

$$\langle W_F \rangle = tr P_B P_F \langle \exp\{ig\int_C B_\mu du_\mu + 2ig\int_0^s \hat{F} d\tau + 2ig\int_0^{s'} \hat{F} d\tau'\}\rangle \qquad (4)$$

Here $P_B, P_F$ are ordering operators of the color matrices $B_\mu$ and $\hat{F}$ respectively. The terms with $\hat{F}$ generate spin–dependent nonperturbative contributions, which are calculable and relatively small, and we shall treat those terms perturbatively.

Neglecting $\hat{F}'s$ as a first approximation, one arrives at the Wilson loop in the adjoint representation, for which one can use the minimal area law, confirmed by numerous lattice data [9] up to the distance of the order of $1fm$ at least,

$$\langle W_{adj} \rangle = Z \ exp(-\sigma_{adj} S_{min}) \tag{5}$$

where we have included in $Z$ self-energy and nonasymptotic corrections, since (5) is valid for large loops with size $R, T \gg T_g$, where $T_g$ is the gluon correlation length.

Applying now the general method of [5] to the Green's function (3), one obtaines the Hamiltonian $H_0$ for 2 gluons without spin corrections.

$$H_0 = \frac{p_r^2}{\mu(t)} + \mu(t) + \frac{L(L+1)}{r^2[\mu + 2\int_0^1 (\beta - \frac{1}{2})^2 \nu d\beta]} + \\ + \int_0^1 \frac{\sigma_{adj}^2 d\beta}{2\nu(\beta,t)} r^2 + \frac{1}{2}\int_0^1 \nu(\beta,t) d\beta \tag{6}$$

Here $\mu(t)$ and $\nu(\beta,t)$ are positive auxiliary functions which are to be found from the extremum condition [5]. Their extremal values are equal to the effective gluon energy $\langle \mu \rangle$ and energy density of the adjoint string $\langle \nu \rangle$.

For the case $L = 0$ the extremization over $\mu$ and $\nu$ yields a simple answer, coinciding with the Hamiltonian of the relativistic potential model

$$H_0 = 2\sqrt{p_r^2} + \sigma_{adj} r \tag{7}$$

With the replacement of the operators $\mu(t), \nu(t,\beta)$ (which by extremization are expressed through operators $p, r$) by $c$ –numbers, to be found from extremization of eigenvalues of $H_0$, one obtains a simple form, used in [8],

$$H_0' = \frac{\mathbf{p}^2}{\mu_0} + \mu_0 + \sigma_{adj} r \tag{8}$$

The eigenvalues of (8) are about 5% higher than those of $H_0$ in eq.(7).

The value of $\sigma_{adj}$ in (8) can be found from the string tension $\sigma_{fund}$ of $q\bar{q}$ system, multiplying it by $\frac{9}{4}$, as it follows from Casimir scaling observed in lattice calculations [9]. Taking experimental Regge slope for mesons $\alpha' = 0.89 \ GeV^{-2}$ one obtains $\sigma_{fund} = 0.18 \ GeV^2$ and $\sigma_{adj} \approx 0.40 \ GeV^2$.

In what follows the parameter $\mu$ and its optimal value $\mu_0$, which is computed from eigenvalues of (8) and proportional to $\sqrt{\sigma_{adj}}$, play very important role. The way they enter spin corrections (see below) and magnetic moments shows that $\mu_0$ is an effective (constituent) gluon mass (or constituent quark mass in the equation for the $q\bar{q}$ system).

Note that our lowest "constituent gluon mass" $\mu_0(n = l = 0) = 0.528\ GeV$ (for $\sigma_f = 0.18\ GeV$) is not far from the values introduced in the potential models, the drastic difference is that our $\mu_0$ is calculable and depends on $n, l$ and grows for higher states.

The mass spectrum of $H_0$ in eq. (6) was studied in refs.[5,6]. With a good accuracy it can be described by a very simple formula

$$\frac{M^2}{2\pi\sigma} = L + 2n_r + c_1 \tag{9}$$

where $L$ is the orbital momentum, $n_r$ –radial quantum number and $c_1$ is a constant $\approx 1.55$. It describes an infinite set of linear Regge-trajectories shifted by $2n_r$ from the leading one ($n_r = 0$). The only difference between light quarks and gluons is the value of $\sigma$, which determines the mass scale. A similar spectrum was obtained independently by numerical quantization of $\bar{q}q$ sysytem in [10].

The lowest glueball state with $L = 0, n_r = 0$ has $M = 2.01GeV$. It corresponds to a degenerate $0^{++}$ and $2^{++}$ state.

In order to compare our results with the corresponding lattice calculations [2,3] it is convenient to consider the quantity $\bar{M}/\sqrt{\sigma_f}$, which is not sensitive to the choice of string tension $\sigma$ [1]. We also introduce the spin averaged mass $\bar{M}$, which for $L = 0, n_r = 0$ states is defined as $\bar{M} = \frac{1}{3}(M(0^{++}) + 2M(2^{++}))$, and in a similar way for higher states.

The comparison of our predictions for spin averaged masses of the lowest glueball states with corresponding lattice results is given in Table 1. For the average mass with $L = 2, n_r = 0$ lattice results are limited to the state $3^{++}$.

It follows from Table 1 that the spin–averaged masses obtained from purely confining force with relativistic kinematics for valence gluons are in a good correspondence with lattice data, which implies that PGE effects are suppressed. We come back to this point in what follows.

Now we shall consider spin splittings of glueball masses and shall treat spin effects as a small perturbation. Such a treatment is justified aposteriori by our results and by lattice data, which demonstrate that spin splittings

[1] Note that the values $\sigma_f \simeq 0.21 - 0.23 GeV^2$ used in lattice calculations differs by about 20% from the "experimental" value $\sigma_f = 0.18 GeV^2$.

for glueball states (apart from $2^{++}-0^{++}$) amount to less than 10-15% of the total mass.

To proceed one should choose among two possibilities of spin state description of bound gluons. One can insist on transversality condition ( valid for a free gluon) also for gluons in the glueball, as it is done in the potential model of ref.[11]. Instead in our formalism the bound gluon becomes massive due to the string ( with the effective energy $\mu_0$ not equal to the gluon momentum $|\mathbf{k}_1|$) and has 3 spin polarizations, similarly to $W^{\pm}, Z^0$ bosons, which get their mass from Higgs condensate. For more discussion see [12].

Then two–gluon mass operator can be written as

$$M = M_0(n,L) + \mathbf{SL}M_{SL} + \mathbf{S}^{(1)}\mathbf{S}^{(2)}M_{SS} + M_T, \tag{10}$$

where $\mathbf{S} = \mathbf{S}^{(1)} + \mathbf{S}^{(2)}$, $M_0$ is the eigenvalue of the Hamiltonian $H \equiv H_0 + \Delta H_{pert}$, and $H_0$ is given in (7) (or its approximation in (8)), while $\Delta H_{pert}$ is due to PGE.

To obtain three other terms in (10) one should consider averaging of the operators $\hat{F}$ in the exponent of (4) and take into account that

$$-2i\hat{F}_{\mu\nu} = 2(\mathbf{S}^{(1)}\mathbf{B}^{(1)} + \tilde{\mathbf{S}}^{(1)}\mathbf{E}^{(1)})_{\mu\nu} \tag{11}$$

and similarly for the term in the integral $\int \hat{F}d\tau'$, with the replacement of indices $1 \to 2$. Here gluon spin operators are introduced, e.g.

$$(S_m^{(1)})_{ik} = -ie_{mik}, \quad i,k = 1,2,3, (\tilde{S}_m^{(1)})_{i4} = -i\delta_{im} \tag{12}$$

Now the spin splittings are obtained from (4) where the proper time $(\tau,\tau')$ is replaced by the real (Euclidean) time $(t,t')$ via the relation [4-6] $d\tau = \frac{dt}{2\mu(t)}$, where $\mu(t)$ is the same operator as in the Hamiltonian (6) with its extremal value $\mu_0$ playing the role of the gluon constituent mass ( and explicitly calculated from (6) or (8)). As a result the spin–dependent part of the Hamiltonian(10) can be written in the form similar to that of Eichten and Feinberg [13] (see [12] for more details)

$$\Delta H_s = \frac{\mathbf{SL}}{\mu_0^2}(\frac{1}{r}\frac{dV_1}{dr} + \frac{1}{r}\frac{dV_2}{dr}) + \frac{\mathbf{S}^{(1)}\mathbf{S}^{(2)}}{3\mu_0^2}V_4(r) +$$

$$+\frac{1}{3\mu_0^2}(3(\mathbf{S}^{(1)}\mathbf{n})(\mathbf{S}^{(2)}\mathbf{n}) - \mathbf{S}^{(1)}\mathbf{S}^{(2)})V_3(r) + \Delta V \tag{13}$$

where $\mathbf{S} = \mathbf{S}^{(1)} + \mathbf{S}^{(2)}, \Delta V$ contains higher cumulant contributions which can be estimated of the order of 10% of the main terms in (13) [4].

The functions $V_i(r)$ are the same as for heavy quarkonia [14] and the NP part of spin splittings can be in this way expressed through the field correlators $D(x), D_1(x)$ [15], measured on the lattice [16]. The resulting estimates show that the only appreciable NP spin–splitting is due to $V_1, V_2$ (Thomas precession) and was computed numerically as in [14].

Perturbative contributions to spin splittings can be calculated in the same way using results of ref.[11]. Combining all corrections and values of $M_0$ from Table 1 we obtain glueball masses compared with lattice data in Table 2 for $\sigma_f = 0.228\ GeV^2$.

The general feature of spin–dependent contribution $\Delta H_s$ is that it dies out fast with the growing orbital or radial number. This feature is well supported by the lattice data in Table 2.

One can see in Table 2 that magnitude of our spin–spin and spin–orbit splittings ( proportional to $\langle\delta^{(3)}(r)\rangle$ and $\langle 1/r^3\rangle$ respectively) is in a good agreement with lattice data; inclusion of PGE in the form of adjoint Coulomb would increase splittings several times [8] and destroy the agreement, yielding another phenomenological argument against adjoint Coulomb form of PGE.

In many analytic calculations of glueball masses it is postulated that there is a Coulomb-type interaction between valence gluons, which differs from the $q\bar{q}$ case by the same $\frac{9}{4}$ factor as in eq. (9). If this would be true it would lead to a drastic decrease of glueball masses calculated above due to Coulomb attraction, especially for L=0 (for $\alpha_s = 0.3$ this mass drops down by 0.5 GeV [8]). This clearly contradicts lattice calculations and casts a doubt on the validity of the assumption about presence of Coulomb interaction (without higher loop corrections).

A careful study of general derivation of Coulomb-like small distance contribution in this formalism shows [12] that contrary to the quark case for valence gluons perturbative gluon exchanges do not sum up to the Coulomb potential.

In order to estimate effects of small distance contributions we shall use the analysis of these effects on gluonic Regge–trajectories not from the glueballs mass spectra at positive $t$, but for $t = 0$. Extensive calculations of the gluonic Pomeron trajectory intercept have been carried out in the leading log approximation (LLA) [17] and $\alpha_s$ cocorrections were calculated recently [18]. An intercept of the leading Regge pole is strongly modified by $\alpha_s$ corrections

$$\Delta = \alpha_P(0) - 1 = \alpha_s \frac{12}{\pi} ln2(1 - C\alpha_s) \tag{14}$$

The coefficient $C$ is large ($\approx 6.5$) and $\alpha_s$ correction substantially decreases

$\Delta$ compared to LLA result. Its value depends on the renormalization scheme and scale for $\alpha_s$. In the "physical" (BLM) scheme values of $\Delta$ are in the region $0.15 \div 0.17$ [19]. We can estimate mass-shift of the lowest glueball state due to PGE effects using this result and assuming that the slope $\alpha'_P \approx 0.4\ GeV^{-2}$ of eq.(11) is not strongly modified by perturbative effects. Thus one can expect that characteristic shift due to perturbative effects in $\bar{M}^2(L=0,n_r=0), \delta\bar{M}^2 \approx \Delta/\alpha'_{adj} \approx (0.38 \div 0.48)\ GeV^2$. This corresponds to the shift in $\bar{M}(l=0,n_r=0), \delta\bar{M} \approx \delta\bar{M}^2/2\bar{M} \approx 0.1\ GeV$. This shift is much smaller than naive estimate using adjoint Coulomb interaction and gives some justification to neglect by PGE in our calculations. It should be noted that this is only a rough estimate of the perturbative effects.

The three–gluon system may be considered in the same way, as it was done for the two-gluon glueballs. The $3g$ Green's function $G^{(3g)}$ is obtained as the background–averaged product of 3 one–gluon Green's function, in full analogy with (1). Assuming large $N_c$ limit for simplicity and neglecting spin splittings one arrives at the path integral (cf equation (3))

$$G^{(3g)} = const \prod_{i=1}^{3} \int_0^\infty ds_i Dz^{(i)} e^{-K_i - \sigma S_i} \tag{15}$$

where $\sigma \equiv \sigma_{fund}$, since every gluon is connected by a fundamental string with each of its neighbours.

Using as before method of refs. [4-6] one obtains the following Hamiltonian (we assume symmetric solution with equal $\mu_i(\tau) \equiv \mu(\tau), i = 1,2,3$ (no orbital excitations was assumed as in (8))

$$H^{(3g)} = \frac{\mathbf{p}_\eta^2 + \mathbf{p}_\xi^2}{2\mu} + \frac{3\mu}{2} + \sigma(|\mathbf{r}_1 - \mathbf{r}_2| + |\mathbf{r}_2 - \mathbf{r}_3|) \tag{16}$$

Here $\mathbf{p}_\eta, \mathbf{p}_\xi$ are Jacobi momenta for the 3–body system .

To simplify treatment further, we shall consider $\mu$ as a constant to be found from the extremum of eigenvalues, as in (8), which in that case provided some 5% increase in eigenvalues and we expect the same situation in this case.

The eigenvalues of $H^{(3g)}$ were found using the hyperspherical method introduced in [20]. A reliable estimate of $M$ gives for the lowest state (and $\sigma_f = 0.18\ GeV^2$) the value $M_0 = 3.22\ GeV$ ($3.62\ GeV$ for $\sigma = 0.23\ GeV^2$). Perturbative hyperfine interaction increases the mass of the $3^{--}$ spin splittings compared to this value by $\approx 0.29\ GeV$. The large value for the lowest three-gluon state is in an agreement with lattice calculations (see Table 2). For more details concerning $3g$ states see [12].

Let us consider now the problem of the Pomeron Regge trajectory in more details, taking into account both nonperturbative and perturbative contributions to the Pomeron dynamics.

The large distance, nonperturbative contribution gives according to Eq.(12) for the leading glueball trajectory ($n_r = 0$ )

$$\alpha_P(t) = -c_1 + \alpha'_P t + 2 \tag{17}$$

with $\alpha'_P = \frac{1}{2\pi\sigma_a}$.

In eq.(16) we took into account spins of "constituent" gluons, but neglected a small nonperturbative spin-spin interactions. For an intercept of this trajectory we obtain $\alpha_P(0) \approx 0.5$, which is substantially below the value found from analysis of high-energy interactions [7] $\alpha_P(0) = 1.1 \div 1.2$.

The perturbative (BFKL) contribution leads to a shift (increase) of the Pomeron intercept by $\approx 0.2$ as it was explained above. The resulting $\alpha_P^{(0)}(0) \approx 0.7$ is still far from experiment.

The most important nonperturbative source, which can lead to an increase of the Pomeron intercept is, in our opinion, the quark-gluon mixing or account of quark-loops in the gluon "medium". In the 1/N -expansion the effect is proportional to $N_f/N_c$, where $N_f$ is the number of light flavours. In the leading approximation of the $1/N_c$-expansion there are 3 Regge-trajectories with vacuum quantum numbers, - $q\bar{q}$-planar trajectories ($\alpha_f$ made of $u\bar{u}$ and $d\bar{d}$ quarks, $\alpha_{f'}$ made of $s\bar{s}$-quarks) and pure gluonic trajectory - $\alpha_G$. The transitions between quarks and gluons $\sim \frac{1}{N_c}$ will lead to a mixing of these trajectories. Note that for a realistic case of $G, f$ and $f'$-trajectories (Fig.1) all 3-trajectories before mixing are close to each other in the small $t$ region. Trajectory of gluonium crosses planar $f$ and $f'$-trajectories in the positive $t$ region ($t < 1\ GeV^2$). In this region mixing between trajectories is essential even for small coupling matrix $g_{ik}(t)$. Lacking calculation of these effects in QCD we will consider them in a semi–phenomenological manner.

Denoting by $\bar{\alpha}_i$ the bare $f, f'$ and $G$–trajectories and introducing the mixing matrix $g_{ik}(t)(i, k = 1, 2, 3)$ we obtain the following equation for determination of resulting trajectories after mixing [12]

$$j^3 - j^2 \sum \bar{\alpha}_i + j(\sum_{i \neq k} \bar{\alpha}_i \bar{\alpha}_k - g_{ik}^2) - \bar{\alpha}_1 \bar{\alpha}_2 \bar{\alpha}_3 + \sum_{i \neq k \neq l} \bar{\alpha}_l g_{ik}^2 - 2 g_{12} g_{13} g_{23} = 0$$

For realistic values of $g_{ik}(t)$ (for details see [12]) the resulting trajectories are shown in Fig. 1 by solid lines. The Pomeron intercept is shifted to the values $\alpha_P(0) > 1$. For $t > 1 GeV^2$ the Pomeron trajectory is very close to

the planar $f$–trajectory, while the second and third vacuum trajectories – to $\alpha_{f'}$ and $\alpha_G$ correspondingly.

Let us consider now the "odderon"–Regge trajectory – the leading gluonic trajectory with negative signature and $C$-parity. Mass of the lowest $3g$ glueball with spin $3^{--}$ corresponding to this trajectory has been estimated above and found to be large $\approx 3.6\ GeV$ in accord with lattice data. The slope $\alpha'_{3g}$ for this trajectory should be equal to the one of $gg$–trajectory [3] and thus the intercept of the nonperturbative glueball "odderon" is very low $\alpha_{3g}(0) \approx -1.6$. Mixing with $q\bar{q}$–trajectories $(\omega, \varphi)$ is much smaller than in the Pomeron case as there is no crossing of the odderon and $(\omega, \varphi)$ trajectories in the small t-region.

The main results of the paper can be summarized as follows. We calculated the 2g and 3g glueball spectrum analytically and compared resulting masses with lattice data, finding a very good agreement. We stress that our spectrum in contrast to all existing theoretical models contains no fitting parameters, and all masses are expressed in terms of the string tension $\sigma$, as it is done also in lattice calculations. A smallness of Coulomb-type contributions to glueball masses is pointed out. The spin splittings of glueball masses were obtained from first perturbative corrections calculated with nonperturbative wave functions. There is a good agreement on spin splittings between our calculations and lattice data as well. This indicates that the main ingredient of the glueball dynamics is the adjoint string occurring between gluons in the two-gluon glueballs. The string dynamics implies that glueball spectrum corresponds to straight-line Regge trajectories, which have the Regge slope equal to $\frac{4}{9}$ of that for meson trajectories. For the Pomeron Regge trajectory we found that with an account of only gluonic contributions (both nonperturbative and perturbative) the intercept is below unity and in order to obtain a phenomenologically exceptable value of the intercept it is necessary to take into account mixing between gluons and $q\bar{q}$ pairs. We note that both nonperturbative (string dynamics, quark loops) and perturbative effects are important to obtain $\alpha_P(0) > 1$. In this approach the "soft" and "hard" dynamics are strongly intermixed to produce the leading Pomeron pole.

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[3] The situation is analogous to the case of $q\bar{q}$(meson) and $qqq$ (baryon) Regge trajectories, since both for baryons and $3g$ glueballs at large $L$ the structure $q - 2q(g - 2g)$ is energetically preferrable.

**Table 1**
Spin averaged glueball masses $M_G/\sqrt{\sigma_f}$

| Quantum numbers | | This work | Lattice data | |
|---|---|---|---|---|
| | | | ref. [3] | ref. [2] |
| 2 gluon states | $l=0, n_r=0$ | 4.68 | 4.66±0.14 | 4.55±0.23 |
| | $l=1, n_r=0$ | 6.0 | 6.36±0.6 | 6.1 ±0.5 |
| | $l=0, n_r=1$ | 7.0 | 6.68±0.6 | 6.45±0.5 |
| | $l=2, n_r=0$ | 7.0 | 9.0 ±0.7($3^{++}$) | 7.7 ±0.4($3^{++}$) |
| | $l=1, n_r=1$ | 8.0 | | 8.14 ±0.4($2^{*-+}$) |
| 3 gluon state | K=0 | 7.61 | | 8.19±0.48 |

**Table 2**
Comparison of predicted glueball masses with lattice data (for $\sigma_f = 0.228 GeV^2$)

| $J^{PC}$ | M(GeV) This work | Lattice data | |
|---|---|---|---|
| | | ref. [2] | ref. [3] |
| $0^{++}$ | 1.58 | 1.73±0.13 | 1.74±0.05 |
| $0^{++*}$ | 2.71 | 2.67±0.31 | 3.14±0.10 |
| $2^{++}$ | 2.59 | 2.40±0.15 | 2.47±0.08 |
| $2^{++*}$ | 3.73 | 3.29±0.16 | 3.21±0.35 |
| $0^{-+}$ | 2.56 | 2.59±0.17 | 2.37±0.27 |
| $0^{-+*}$ | 3.77 | 3.64±0.24 | |
| $2^{-+}$ | 3.03 | 3.1±0.18 | 3.37±0.31 |
| $2^{-+*}$ | 4.15 | 3.89±0.23 | |
| $3^{++}$ | 3.58 | 3.69±0.22 | 4.3±0.34 |
| $1^{--}$ | 3.49 | 3.85±0.24 | |
| $2^{--}$ | 3.71 | 3.93±0.23 | |
| $3^{--}$ | 4.03 | 4.13±0.29 | |

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Figure 1 Liang et al. PRB

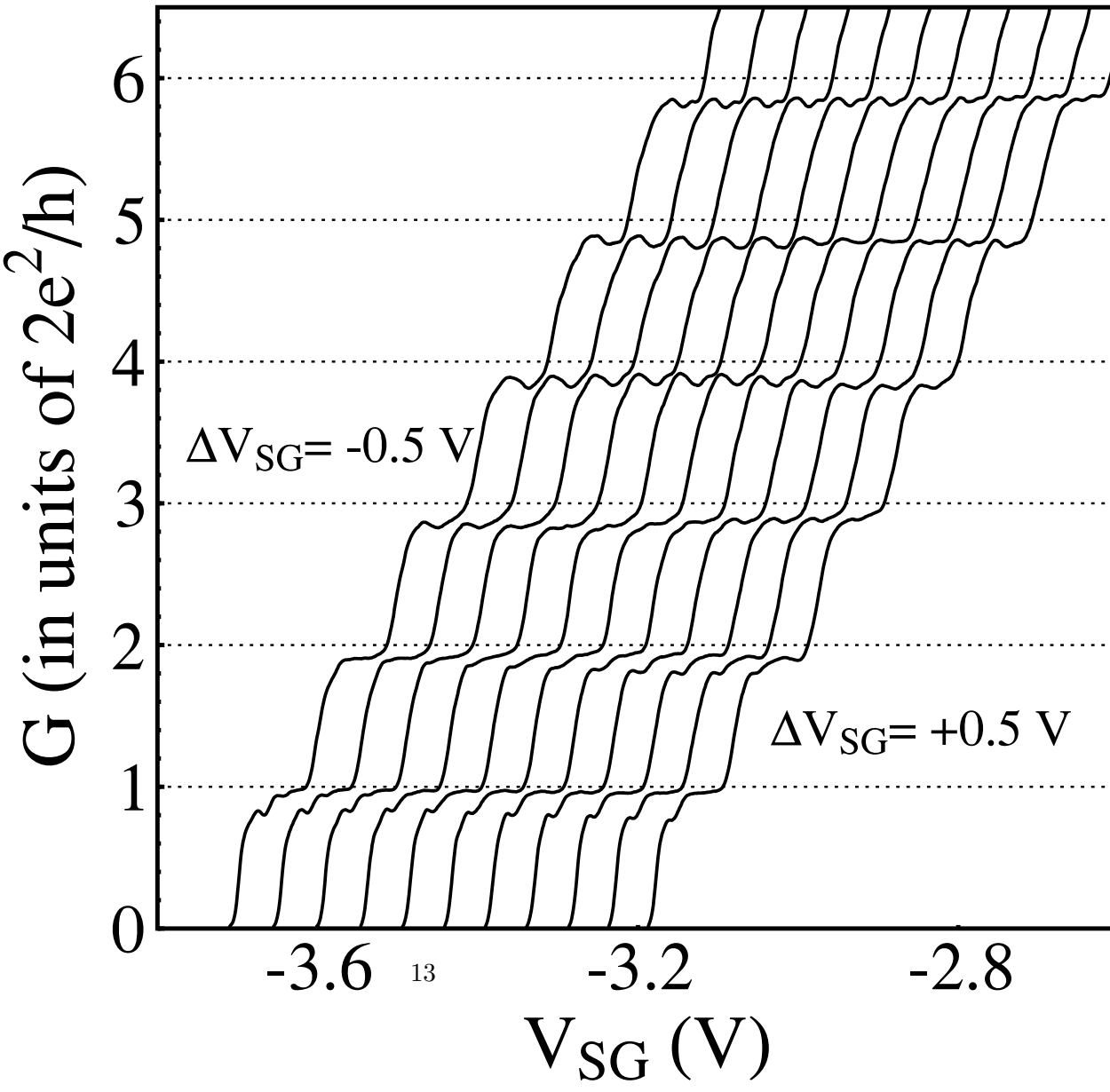


Figure 1: Vacuum trajectories before mixing (dotted lines) and after mixing (solid lines).